

## Factorising complete squares

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The technique of factorising a quadratic expression has been explained on leaflet *Factorising quadratic expressions*. There is a special case of quadratic expression known as a **complete square**. This leaflet explains what this means and how such expressions are factorised.

### What is meant by a complete square ?

A quadratic expression is called a **complete square** when it can be written in the form  $(\quad)^2$ , that is as a single term, squared.

Consider the following example.

#### Example

Factorise  $x^2 + 10x + 25$ .

We write

$$x^2 + 10x + 25 = (x \quad)(x \quad)$$

and seek two numbers which add to give 10 and multiply to give 25. The two required numbers are 5 and 5 and so

$$x^2 + 10x + 25 = (x + 5)(x + 5)$$

Because both brackets are the same the result can be written as  $(x + 5)^2$ . This is a single term, squared, - that is, a **complete square**.

#### Example

Factorise  $x^2 - 8x + 16$ .

Proceeding as before, we write

$$x^2 - 8x + 16 = (x \quad)(x \quad)$$

and seek two numbers which add to give  $-8$  and multiply to give 16. The two required numbers are  $-4$  and  $-4$  and so

$$x^2 - 8x + 16 = (x - 4)(x - 4)$$

The result can be written as  $(x - 4)^2$ , a **complete square**.

More complicated examples can occur, for example when there is a number in front of the  $x^2$ . Work through the following example.

### Example

Factorise  $25x^2 - 20x + 4$ .

Note that  $25x^2$  can be written as  $(5x)^2$ , a squared term. Note also that  $4 = 2^2$ . In this case, by inspection,

$$25x^2 - 20x + 4 = (5x - 2)(5x - 2)$$

The result can be written as  $(5x - 2)^2$ , a **complete square**.

Do not worry if you have difficulty with this last example. The skill will come with practice.

### Exercises

1. Factorise the following.

- a)  $x^2 + 18x + 81$     b)  $x^2 - 4x + 4$     c)  $x^2 - 22x + 121$   
d)  $25x^2 + 40x + 16$     e)  $64x^2 + 16x + 1$

### Answers

1. a)  $(x + 9)^2$     b)  $(x - 2)^2$     c)  $(x - 11)^2$   
d)  $(5x + 4)^2$     e)  $(8x + 1)^2$